O K L A H O M A   S T A T E   U N I V E R S I T Y

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECEN 5713 Linear Systems
Fall 2002
Midterm Exam #2

DO ALL FIVE PROBLEMS

Name : ______________________________

Student ID: ________________________________

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<u>Problem 1</u>:

Consider $Ax = y$, where A is $m \times n$ and has rank m. Is $(A^T A)^{-1} A^T y$ a solution? If not, under what condition will it be a solution? Is $A^T (AA^T)^{-1} y$ a solution?

Problem 2:
Consider the linear operator

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & -2 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

determine its rank and nullity, then find a basis for the range space and the null space of the linear operator, A, respectively ?

Problem 3:

Let V and W be vector space over the same field F and let $A: V \to W$ be a linear transformation. Show that A is a one-to-one mapping if and only if null space of A, $N(A) = \{0\}$.

Problem 4:

Consider the set of all 2×2 matrices in the form

$$\begin{bmatrix} y & x \\ x & -y \end{bmatrix}$$

where x and y are arbitrary real numbers (i.e., $x,\ y \in \Re$). Does the set with the usual definitions of matrix addition and multiplication form a field ? If not, show why ? If yes, what are the zero and unity elements ?

Problem 5:

Show if the following sets

$$\begin{bmatrix} 2 \\ 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -1 \\ 2 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 1 \\ 1 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ -1 \\ -1 \end{bmatrix}$$

span the same subspace V of $\left(\Re^4, \Re\right)$.